

FINAL JEE-MAIN EXAMINATION – JUNE, 2022

 (Held On Tuesday 28th June, 2022)

TIME : 3 : 00 PM to 06 : 00 PM

MATHEMATICS

SECTION-A

1. Let $R_1 = \{(a, b) \in \mathbb{N} \times \mathbb{N} : |a - b| \leq 13\}$ and $R_2 = \{(a, b) \in \mathbb{N} \times \mathbb{N} : |a - b| \neq 13\}$. Then on $\mathbb{N}$:

- (A) Both R_1 and R_2 are equivalence relations
 (B) Neither R_1 nor R_2 is an equivalence relation
 (C) R_1 is an equivalence relation but R_2 is not
 (D) R_2 is an equivalence relation but R_1 is not

Official Ans. by NTA (B)

Allen Overseas Ans. (B)

- Sol.** $R_1 = \{(a, b) \in \mathbb{N} \times \mathbb{N} : |a - b| \leq 13\}$
 $R_2 = \{(a, b) \in \mathbb{N} \times \mathbb{N} : |a - b| \neq 13\}$.
 For R_1 :

- i) Reflexive relation
 $(a, a) \in \mathbb{N} \times \mathbb{N} : |a - a| \leq 13$
 ii) Symmetric relation
 $(a, b) \in R_1, (b, a) \in R_1 : |b - a| \leq 13$
 iii) Transitive relation
 $(a, b) \in R_1, (b, c) \in R_1, (a, c) \in R_1 :$
 $(1, 3) \in R_1, (3, 16) \in R_1$ but $(1, 16) \notin R_1$

For R_2 :

- i) Reflexive relation
 $(a, a) \in \mathbb{N} \times \mathbb{N} : |a - a| \neq 13$
 ii) Symmetric relation
 $(b, a) \in \mathbb{N} \times \mathbb{N} : |b - a| \neq 13$
 iii) Transitive relation
 $(a, b) \in R_2, (b, c) \in R_2, (a, c) \in R_2$

$(1, 3) \in R_2, (3, 14) \in R_2$ but $(1, 14) \notin R_2$

2. Let $f(x)$ be a quadratic polynomial such that $f(-2) + f(3) = 0$. If one of the roots of $f(x) = 0$ is -1 , then the sum of the roots of $f(x) = 0$ is equal to :

- (A) $\frac{11}{3}$ (B) $\frac{7}{3}$
 (C) $\frac{13}{3}$ (D) $\frac{14}{3}$

Official Ans. by NTA (A)

Allen Overseas Ans. (A)

TEST PAPER WITH SOLUTION

Sol. $f(-2) + f(3) = 0$

$$f(x) = (x + 1)(ax + b)$$

$$f(-2) + f(3) = -1(-2a + b) + 4(3a + b) = 0$$

$$2a - b + 12a + 4b = 0$$

$$14a + 3b = 0$$

$$\frac{-b}{a} = \frac{14}{3}$$

$$\text{Sum of roots} = \left(-1 + \frac{-b}{a}\right) = -1 + \frac{14}{3} = \frac{11}{3}$$

3. The number of ways to distribute 30 identical candies among four children C_1, C_2, C_3 and C_4 so that C_2 receives atleast 4 and atmost 7 candies, C_3 receives atleast 2 and atmost 6 candies, is equal to

- (A) 205 (B) 615
 (C) 510 (D) 430

Official Ans. by NTA (D)

Allen Overseas Ans. (D)

Sol. $t_1 + t_2 + t_3 + t_4 = 30$

$$\text{Coefficient of } x^{30} \text{ in } (1 + x + x^2 + \dots + x^{30})^2$$

$$(x^4 + x^5 + x^6 + x^7)(x^2 + x^3 + x^4 + x^5 + x^6)$$

$$x^6 \left(\frac{1 - x^{31}}{1 - x} \right)^2 (1 + x + x^2 + x^3)(1 + x + x^2 + x^3 + x^4)$$

$$x^6 (1 - x^{31})^2 (1 - x^4)(1 - x^5)(1 - x)^4$$

$$x^6 (1 - x^4 - x^5 + x^9)(1 + x^{62} - 2x^{31}(1 - x)^{-4})$$

$$x^6 (1 - x^4 - x^5 + x^9)(1 - x)^{-4}$$

$$\text{Coefficient of } x^n \text{ in } (1 - x)^{-r} \text{ is } {}^{n+r-1}C_{r-1}$$

$$\Rightarrow {}^{27}C_3 - {}^{23}C_3 - {}^{22}C_3 + {}^{18}C_3$$

$$2925 - 1771 - 1540 + 816$$

$$= 430$$

OR

$$x_2 \in [4, 7], x_3 \in [2, 6]$$

$$\Rightarrow t_1 + t_2 + t_3 + t_4 = 24$$

$$\text{total ways} =$$

$${}^{24+4-1}C_{4-1} - {}^{20+4-1}C_{4-1} - {}^{19+4-1}C_{4-1} + {}^{15+4-1}C_{4-1}$$

$$= {}^{27}C_3 - {}^{23}C_3 - {}^{22}C_3 + {}^{18}C_3 = 430$$

4. The term independent of x in the expression of

$$\left(1 - x^2 + 3x^3\right)\left(\frac{5}{2}x^3 - \frac{1}{5x^2}\right)^{11}, x \neq 0 \text{ is}$$

- (A) $\frac{7}{40}$ (B) $\frac{33}{200}$
(C) $\frac{39}{200}$ (D) $\frac{11}{50}$

Official Ans. by NTA (B)

Allen Overseas Ans. (B)

Sol. $\left(1 - x^2 + 3x^3\right)\left(\frac{5}{2}x^3 - \frac{1}{5x^2}\right)^{11}$

General term of $\left(\frac{5}{2}x^3 - \frac{1}{5x^2}\right)^{11}$ is

$${}^{11}C_r \left(\frac{5}{2}x^3\right)^{11-r} \left(-\frac{1}{5x^2}\right)^r$$

General term is ${}^{11}C_r \left(\frac{5}{2}\right)^{11-r} \left(-\frac{1}{5}\right)^r x^{33-5r}$

Now, term independent of x

$$1 \times \text{coefficient of } x^0 \text{ in } \left(\frac{5}{2}x^3 - \frac{1}{5x^2}\right)^{11}$$

$$-1 \times \text{coefficient of } x^{-2} \text{ in } \left(\frac{5}{2}x^3 - \frac{1}{5x^2}\right)^{11} +$$

$$3 \times \text{coefficient of } x^{-3} \text{ in } \left(\frac{5}{2}x^3 - \frac{1}{5x^2}\right)^{11}$$

for coefficient of x^0 $33 - 5r = 0$ not possible

for coefficient of x^{-2} $33 - 5r = -2$

$$35 = 5r \Rightarrow r = 7$$

for coefficient of x^{-3} $33 - 5r = -3$

$$36 = 5r \text{ not possible}$$

So term independent of x is

$$(-1) {}^{11}C_7 \left(\frac{5}{2}\right)^4 \left(-\frac{1}{5}\right)^7 = \frac{33}{200}$$

5. If n arithmetic means are inserted between a and 100 such that the ratio of the first mean to the last mean is $1 : 7$ and $a + n = 33$, then the value of n is

- (A) 21 (B) 22
(C) 23 (D) 24

Official Ans. by NTA (C)

Allen Overseas Ans. (C)

Sol. $d = \frac{100 - a}{n + 1}$

$$A_1 = a + d$$

$$A_n = 100 - d$$

$$\Rightarrow \frac{A_1}{A_n} = \frac{1}{7} \Rightarrow \frac{a + d}{100 - d} = \frac{1}{7}$$

$$\Rightarrow 7a + 8d = 100$$

$$\Rightarrow 7a + 8\left(\frac{100 - a}{n + 1}\right) = 100 \quad \dots(1)$$

$$\therefore a + n = 33 \quad \dots(2)$$

Now, by Eq. (1) and (2)

$$7n^2 - 132n - 667 = 0$$

$$\boxed{n = 23} \text{ and } n = \frac{-29}{7} \text{ reject.}$$

6. Let $f, g : \mathbf{R} \rightarrow \mathbf{R}$ be functions defined by

$$f(x) = \begin{cases} [x] & , x < 0 \\ |1 - x| & , x \geq 0 \end{cases} \text{ and}$$

$$g(x) = \begin{cases} e^x - x & , x < 0 \\ (x - 1)^2 - 1 & , x \geq 0 \end{cases}$$

where $[x]$ denote the greatest integer less than or equal to x . Then, the function $f \circ g$ is discontinuous at exactly :

- (A) one point (B) two points
(C) three points (D) four points

Official Ans. by NTA (B)

Allen Overseas Ans. (B)

- Sol.** Check continuity at $x = 0$ and also check continuity at those x where $g(x) = 0$

$$g(x) = 0 \text{ at } x = 0, 2$$

$$f \circ g(0^+) = -1$$

$$f \circ g(0) = 0$$

Hence, discontinuous at $x = 0$

$$f \circ g(2^+) = 1$$

$$f \circ g(2^-) = -1$$

Hence, discontinuous at $x = 2$

7. Let $f: \mathbf{R} \rightarrow \mathbf{R}$ be a differentiable function such that $f\left(\frac{\pi}{4}\right) = \sqrt{2}$, $f\left(\frac{\pi}{2}\right) = 0$ and $f'\left(\frac{\pi}{2}\right) = 1$ and

$$\text{let } g(x) = \int_x^{\pi/4} (f'(t) \sec t + \tan t \sec t f(t)) dt \text{ for}$$

$$x \in \left[\frac{\pi}{4}, \frac{\pi}{2}\right]. \text{ Then } \lim_{x \rightarrow \left(\frac{\pi}{2}\right)^-} g(x) \text{ is equal to}$$

- (A) 2 (B) 3
(C) 4 (D) -3

Official Ans. by NTA (B)

Allen Overseas Ans. (B)

Sol. $g(x) = \int_x^{\pi/4} (f'(t) \sec t + \tan t \sec t f(t)) dt$

$$g(x) = \int_x^{\pi/4} d(f(t) \cdot \sec t) = f(t) \sec t \Big|_x^{\pi/4}$$

$$g(x) = f\left(\frac{\pi}{4}\right) \sec \frac{\pi}{4} - f(x) \cdot \sec x$$

$$g(x) = 2 - f(x) \sec x = 2 - \left(\frac{f(x)}{\cos x}\right)$$

$$\lim_{x \rightarrow \left(\frac{\pi}{2}\right)^-} g(x) = 2 - \lim_{x \rightarrow \left(\frac{\pi}{2}\right)^-} \left(\frac{f(x)}{\cos x}\right)$$

Using L'Hopital Rule

$$= 2 - \lim_{x \rightarrow \left(\frac{\pi}{2}\right)^-} \frac{f'(x)}{(-\sin x)}$$

$$= 2 + \frac{f'\left(\frac{\pi}{2}\right)}{\sin \frac{\pi}{2}} = 2 + \frac{1}{1} = 3$$

8. Let $f: \mathbf{R} \rightarrow \mathbf{R}$ be continuous function satisfying $f(x) + f(x+k) = n$, for all $x \in \mathbf{R}$ where $k > 0$ and n

is a positive integer. If $I_1 = \int_0^{4nk} f(x) dx$ and

$$I_2 = \int_{-k}^{3k} f(x) dx, \text{ then}$$

- (A) $I_1 + 2I_2 = 4nk$ (B) $I_1 + 2I_2 = 2nk$
(C) $I_1 + nI_2 = 4n^2k$ (D) $I_1 + nI_2 = 6n^2k$

Official Ans. by NTA (C)

Allen Overseas Ans. (C)

Sol. $f(x) + f(x+k) = n$

$$\Rightarrow f(x) = f(x+2k)$$

$f(x)$ is periodic with period $2k$

$$I_1 = \int_0^{4nk} f(x) dx = 2n \int_0^{2k} f(x) dx$$

$$I_2 = \int_{-k}^{3k} f(x) dx = 2 \int_0^k f(x) dx$$

Now,

$$f(x) + f(x+k) = n$$

$$\Rightarrow \int_0^k f(x) dx + \int_0^k f(x+k) dx = nk$$

$$\Rightarrow \int_0^k f(x) dx + \int_k^{2k} f(x) dx = nk$$

$$\Rightarrow \int_0^{2k} f(x) dx = nk$$

$$\Rightarrow I_1 = 2n^2k, I_2 = 2nk$$

$$\Rightarrow I_1 + nI_2 = 4n^2k$$

9. The area of the bounded region enclosed by the

curve $y = 3 - \left|x - \frac{1}{2}\right| - |x+1|$ and the x-axis is

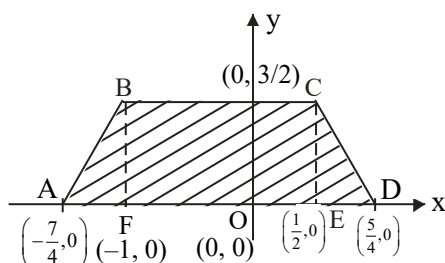
- (A) $\frac{9}{4}$ (B) $\frac{45}{16}$
(C) $\frac{27}{8}$ (D) $\frac{63}{16}$

Official Ans. by NTA (C)

Allen Overseas Ans. (C)

Sol. $y = \begin{cases} 3 + (x+1) + \left(x - \frac{1}{2}\right), & x < -1 \\ 3 - (x+1) + \left(x - \frac{1}{2}\right), & -1 \leq x < \frac{1}{2} \\ 3 - (x+1) - \left(x - \frac{1}{2}\right), & \frac{1}{2} \leq x \end{cases}$

$$y = \begin{cases} \frac{7}{2} + 2x, & x < -1 \\ \frac{3}{2}, & -1 \leq x < \frac{1}{2} \\ \frac{5}{2} - 2x, & \frac{1}{2} \leq x \end{cases}$$



Area bounded = ar ABF + ar BCEF + ar CDE

$$= \frac{1}{2} \left(\frac{3}{4} \right) \left(\frac{3}{2} \right) + \left(\frac{3}{2} \right) \left(\frac{3}{2} \right) + \frac{1}{2} \left(\frac{3}{4} \right) \left(\frac{3}{2} \right) = \frac{27}{8} \text{ sq. units.}$$

10. Let $x = x(y)$ be the solution of the differential equation $2ye^{x/y^2} dx + (y^2 - 4xe^{x/y^2}) dy = 0$ such that $x(1) = 0$. Then, $x(e)$ is equal to

- (A) $e \log_e(2)$ (B) $-e \log_e(2)$
(C) $e^2 \log_e(2)$ (D) $-e^2 \log_e(2)$

Official Ans. by NTA (D)

Allen Overseas Ans. (D)

Sol. $2ye^{x/y^2} dx + (y^2 - 4xe^{x/y^2}) dy = 0$

$$2e^{x/y^2} [ydx - 2xdy] + y^2 dy = 0$$

$$2e^{x/y^2} \left[\frac{y^2 dx - x \cdot (2y) dy}{y} \right] + y^2 dy = 0$$

Divide by y^3

$$2e^{x/y^2} \left[\frac{y^2 dx - x \cdot (2y) dy}{y^4} \right] + \frac{1}{y} dy = 0$$

$$2e^{x/y^2} d\left(\frac{x}{y^2}\right) + \frac{1}{y} dy = 0$$

Integrating

$$\int 2e^{x/y^2} d\left(\frac{x}{y^2}\right) + \int \frac{1}{y} dy = 0$$

$$2e^{x/y^2} + \ln y + c = 0$$

(0, 1) lies on it.

$$2e^0 + \ln 1 + c = 0 \Rightarrow c = -2$$

Required curve : $2e^{x/y^2} + \ln y - 2 = 0$

For $x(e)$

$$2e^{x/e^2} + \ln e - 2 = 0 \Rightarrow x = -e^2 \log_e 2$$

11. Let the slope of the tangent to a curve $y = f(x)$ at (x, y) be given by $2 \tan x (\cos x - y)$. if the curve passes through the point $(\pi/4, 0)$, then the value

of $\int_0^{\pi/2} y dx$ is equal to

- (A) $(2 - \sqrt{2}) + \frac{\pi}{\sqrt{2}}$ (B) $2 - \frac{\pi}{\sqrt{2}}$
(C) $(2 + \sqrt{2}) + \frac{\pi}{\sqrt{2}}$ (D) $2 + \frac{\pi}{\sqrt{2}}$

Official Ans. by NTA (B)

Allen Overseas Ans. (B)

Sol. $\frac{dy}{dx} = 2 \tan x \cos x - 2 \tan x \cdot y$

$$\frac{dy}{dx} + (2 \tan x) y = 2 \sin x$$

Integrating factor = $e^{\int 2 \tan x dx} = \frac{1}{\cos^2 x}$

$$y \left(\frac{1}{\cos^2 x} \right) = \int \frac{2 \sin x}{\cos^2 x} dx$$

$$y \sec^2 x = \frac{2}{\cos x} + C$$

$$y = 2 \cos x + C \cos^2 x$$

$$\text{Passes through } \left(\frac{\pi}{4}, 0 \right)$$

$$0 = \sqrt{2} + \frac{C}{2} \Rightarrow C = -2\sqrt{2}$$

$$f(x) = 2 \cos x - 2\sqrt{2} \cos^2 x : \text{Required curve}$$

$$\int_0^{\pi/2} y dx = 2 \int_0^{\pi/2} \cos x dx - 2\sqrt{2} \int_0^{\pi/2} \cos^2 x dx$$

$$= [2 \sin x]_0^{\pi/2} - 2\sqrt{2} \left[\frac{x}{2} + \frac{\sin 2x}{4} \right]_0^{\pi/2}$$

$$= 2 - \frac{\pi}{\sqrt{2}}$$

12. Let a triangle be bounded by the lines $L_1 : 2x + 5y = 10$; $L_2 : -4x + 3y = 12$ and the line L_3 , which passes through the point $P(2, 3)$, intersect L_2 at A and L_1 at B. If the point P divides the line-segment AB internally in the ratio 1 : 3, then the area of the triangle is equal to

(A) $\frac{110}{13}$

(B) $\frac{132}{13}$

(C) $\frac{142}{13}$

(D) $\frac{151}{13}$

Official Ans. by NTA (B)

Allen Overseas Ans. (B)

Sol. Points A lies on L_2

$$A \left(\alpha, 4 + \frac{4}{3}\alpha \right)$$

Points B lies on L_1

$$B \left(\beta, 2 - \frac{2}{5}\beta \right)$$

Points P divides AB internally in the ratio 1 : 3

$$\Rightarrow P(2, 3) = P \left(\frac{3\alpha + \beta}{4}, \frac{3 \left(4 + \frac{4}{3}\alpha \right) + 1 \left(2 - \frac{2}{5}\beta \right)}{4} \right)$$

$$\Rightarrow \alpha = \frac{3}{13}, \beta = \frac{95}{13}$$

$$\text{Point } A \left(\frac{3}{13}, \frac{56}{13} \right), B \left(\frac{95}{13}, -\frac{12}{13} \right)$$

Vertex C of triangle is the point of intersection of L_1 & L_2

$$\Rightarrow C \left(-\frac{15}{13}, \frac{32}{13} \right)$$

$$\text{area } \Delta ABC = \frac{1}{2} \begin{vmatrix} \frac{3}{13} & \frac{56}{13} & 1 \\ \frac{95}{13} & -\frac{12}{13} & 1 \\ -\frac{15}{13} & \frac{32}{13} & 1 \end{vmatrix}$$

$$= \frac{1}{2 \times 13^3} \begin{vmatrix} 3 & 56 & 13 \\ 95 & -12 & 13 \\ -15 & 32 & 13 \end{vmatrix}$$

$$\text{area } \Delta ABC = \frac{132}{13} \text{ sq. units.}$$

13. Let $a > 0$, $b > 0$. Let e and ℓ respectively be the eccentricity and length of the latus rectum of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. Let e' and ℓ' respectively

the eccentricity and length of the latus rectum of its conjugate hyperbola. If $e^2 = \frac{11}{14}\ell$ and $(e')^2 = \frac{11}{8}\ell'$,

then the value of $77a + 44b$ is equal to

(A) 100

(B) 110

(C) 120

(D) 130

Official Ans. by NTA (D)

Allen Overseas Ans. (D)

$$\text{Sol. } e = \sqrt{1 + \frac{b^2}{a^2}}, \ell = \frac{2b^2}{a}$$

$$\text{Given } e^2 = \frac{11}{14}\ell$$

$$1 + \frac{b^2}{a^2} = \frac{11}{14} \cdot \frac{2b^2}{a}$$

$$\frac{a^2 + b^2}{a^2} = \frac{11}{7} \cdot \frac{b^2}{a} \quad \dots\dots(1)$$

Also $e' = \sqrt{1 + \frac{a^2}{b^2}}$, $\ell' = \frac{2a^2}{b}$

Given $(e')^2 = \frac{11}{8} \ell'$

$$1 + \frac{a^2}{b^2} = \frac{11}{8} \cdot \frac{2a^2}{b}$$

$$\frac{a^2 + b^2}{b^2} = \frac{11}{4} \cdot \frac{a^2}{b} \quad \dots\dots(2)$$

New (1) $\div$ (2)

$$\frac{b^2}{a^2} = \frac{4}{7} \cdot \frac{b^3}{a^3}$$

$$\therefore 7a = 4b \quad \dots\dots (3)$$

From (2)

$$\frac{16b^2}{49} + b^2 = \frac{11}{4} \cdot \frac{16b^2}{49b}$$

$$\frac{65}{49} = \frac{11}{4} \cdot \frac{16}{49} \cdot b$$

$$\therefore b = \frac{4 \times 65}{11 \times 16} \quad \dots\dots (4)$$

We have to find value of

$$77a + 44b$$

$$11(7a + 4b) = 11(4b + 4b) = 11 \times 8b$$

$$\therefore \text{Value of } 11 \times 8b = 11 \times 8 \times \frac{4 \times 65}{16 \times 11} = 130$$

14. Let $\vec{a} = \alpha\hat{i} + 2\hat{j} - \hat{k}$ and $\vec{b} = -2\hat{i} + \alpha\hat{j} + \hat{k}$, where $\alpha \in \mathbf{R}$. If the area of the parallelogram whose adjacent sides are represented by the vectors $\vec{a}$ and $\vec{b}$ is $\sqrt{15(\alpha^2 + 4)}$, then the value of

$$2|\vec{a}|^2 + (\vec{a} \cdot \vec{b})|\vec{b}|^2 \text{ is equal to}$$

- (A) 10 (B) 7
(C) 9 (D) 14

Official Ans. by NTA (D)

Allen Overseas Ans. (D)

Sol. $\vec{a} = \alpha\hat{i} + 2\hat{j} - \hat{k}$, $\vec{b} = -2\hat{i} + \alpha\hat{j} + \hat{k}$,
area of parallelogram $= |\hat{a} \times \hat{b}|$

$$|\hat{a} \times \hat{b}| = \sqrt{(\alpha + 2)^2 + (\alpha - 2)^2 + (\alpha^2 + 4)^2}$$

Given $|\hat{a} \times \hat{b}| = \sqrt{15(\alpha^2 + 4)}$

$$2(\alpha^2 + 4) + (\alpha^2 + 4)^2 = 15(\alpha^2 + 4)$$

$$(\alpha^2 + 4)^2 = 13(\alpha^2 + 4)$$

$$\Rightarrow \alpha^2 + 4 = 13 \therefore \alpha^2 = 9$$

$$2|\vec{a}|^2 + (\vec{a} \cdot \vec{b})|\vec{b}|^2$$

$$|\vec{a}|^2 = \alpha^2 + 4 + 1 = \alpha^2 + 5$$

$$|\vec{b}|^2 = 4 + \alpha^2 + 1 = \alpha^2 + 5$$

$$\vec{a} \cdot \vec{b} = -2\alpha + 2\alpha - 1 = -1$$

$$\therefore 2|\vec{a}|^2 + (\vec{a} \cdot \vec{b})|\vec{b}|^2$$

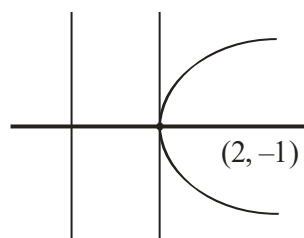
$$2(\alpha^2 + 5) - 1(\alpha^2 + 5) = \alpha^2 + 5 = 14$$

15. If vertex of a parabola is $(2, -1)$ and the equation of its directrix is $4x - 3y = 21$, then the length of its latus rectum is

- (A) 2 (B) 8
(C) 12 (D) 16

Official Ans. by NTA (B)

Allen Overseas Ans. (B)



Sol. $4x - 3y = 21$

$$a = \frac{|8 + 3 - 21|}{5} = \frac{10}{5} = 2$$

$$\therefore \text{latus rectum} = 4a = 8$$

16. Let the plane $ax + by + cz = d$ pass through $(2, 3, -5)$ and is perpendicular to the planes $2x + y - 5z = 10$ and $3x + 5y - 7z = 12$.

If a, b, c, d are integers $d > 0$ and $\gcd(|a|, |b|, |c|, d) = 1$, then the value of $a + 7b + c + 20d$ is equal to

- (A) 18 (B) 20
(C) 24 (D) 22

Official Ans. by NTA (D)
Allen Overseas Ans. (D)

Sol. DR'S normal of plane

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -5 \\ 3 & 5 & -7 \end{vmatrix} = 18\hat{i} - \hat{j} + 7\hat{k}$$

$\therefore$ eqⁿ of plane

$$18x - y + 7z = d$$

It passes through (2, 3, -5)

$$36 - 3 - 35 = d \quad \therefore d = -2$$

$\therefore$ Eqⁿ of plane

$$18x - y + 7z = -2$$

$$-18x + y - 7z = 2$$

$$\therefore a = -18, b = 1, c = -7, d = 2$$

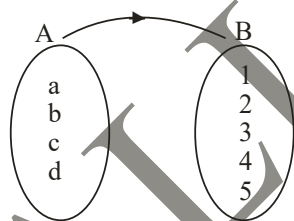
$$a + 7b + c + 20d = -18 + 7 - 7 + 40 = 22$$

17. The probability that a randomly chosen one-one function from the set {a, b, c, d} to the set {1, 2, 3, 4, 5} satisfies $f(a) + 2f(b) - f(c) = f(d)$ is :

- (A) $\frac{1}{24}$ (B) $\frac{1}{40}$
(C) $\frac{1}{30}$ (D) $\frac{1}{20}$

Official Ans. by NTA (D)

Allen Overseas Ans. (D)



Sol.

$$n(s) = {}^5C_4 \times 4! = 120$$

f(a)	+ 2f(b)	=	f(c)	+ f(d)
5	2×1	3	4	
4	2×2	3	5	
1	2×3	2	5	

$$n(A) = 2 \times 3 = 6$$

$$\therefore P(A) = \frac{n(A)}{n(s)} = \frac{6}{120} = \frac{1}{20}$$

18. The value of $\lim_{n \rightarrow \infty} 6 \tan \left\{ \sum_{r=1}^n \tan^{-1} \left(\frac{1}{r^2 + 3r + 3} \right) \right\}$

is equal to

- (A) 1 (B) 2
(C) 3 (D) 6

Official Ans. by NTA (C)

Allen Overseas Ans. (C)

Sol. $T_r = \tan^{-1} \left[\frac{(r+2) - (r+1)}{1 + (r+2)(r+1)} \right]$

$$= \tan^{-1}(r+2) - \tan^{-1}(r+1)$$

$$T_1 = \tan^{-1} 3 - \tan^{-1} 2$$

$$T_2 = \tan^{-1} 4 - \tan^{-1} 3$$

$$T_n = \tan^{-1}(n+2) - \tan^{-1}(n+1)$$

$$S_n = \tan^{-1}(n+2) - \tan^{-1} 2 = \tan^{-1} \left(\frac{n+2-2}{1+2(n+2)} \right)$$

$$= \tan^{-1} \left(\frac{n}{2n+5} \right)$$

$$\lim_{n \rightarrow \infty} 6 \tan \left(\tan^{-1} \left(\frac{n}{2n+5} \right) \right)$$

$$= \lim_{n \rightarrow \infty} \frac{6n}{2n+5} = \frac{6}{2} = 3$$

19. Let $\vec{a}$ be a vector which is perpendicular to the vector

$$3\hat{i} + \frac{1}{2}\hat{j} + 2\hat{k}. \text{ If } \vec{a} \times (2\hat{i} + \hat{k}) = 2\hat{i} - 13\hat{j} - 4\hat{k}, \text{ then}$$

the projection of the vector $\vec{a}$ on the vector

$$2\hat{i} + 2\hat{j} + \hat{k} \text{ is}$$

- (A) $\frac{1}{3}$ (B) 1
(C) $\frac{5}{3}$ (D) $\frac{7}{3}$

Official Ans. by NTA (C)

Allen Overseas Ans. (C)

Sol. $(\vec{a} \times (2\hat{i} + \hat{k})) \times \left(3\hat{i} + \frac{1}{2}\hat{j} + 2\hat{k} \right)$
 $= (2\hat{i} - 13\hat{j} - 4\hat{k}) \times \left(3\hat{i} + \frac{1}{2}\hat{j} + 2\hat{k} \right)$

$$-(6+2)\vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -13 & -4 \\ 3 & \frac{1}{2} & 2 \end{vmatrix}$$

$$\vec{a} = 3\hat{i} + 2\hat{j} - 5\hat{k}$$

Projection of $\vec{a}$ on vector $2\hat{i} + 2\hat{j} + \hat{k}$ is

$$\vec{a} \cdot \frac{(2\hat{i} + 2\hat{j} + \hat{k})}{3} = \frac{5}{3}$$

20. If $\cot \alpha = 1$ and $\sec \beta = -\frac{5}{3}$, where $\pi < \alpha < \frac{3\pi}{2}$

and $\frac{\pi}{2} < \beta < \pi$, then the value of $\tan(\alpha + \beta)$ and

the quadrant in which $\alpha + \beta$ lies, respectively are

(A) $-\frac{1}{7}$ and IVth quadrant

(B) 7 and Ist quadrant

(C) -7 and IVth quadrant

(D) $\frac{1}{7}$ and Ist quadrant

Official Ans. by NTA (A)

Allen Overseas Ans. (A)

Sol. $\cot \alpha = 1, \sec \beta = -\frac{5}{3}, \cos \beta = -\frac{3}{5}, \tan \beta = -\frac{4}{3}$

$$\tan(\alpha + \beta) = \frac{1 - \frac{4}{3}}{1 + \frac{4}{3} \times 1} = \frac{-1}{7}$$

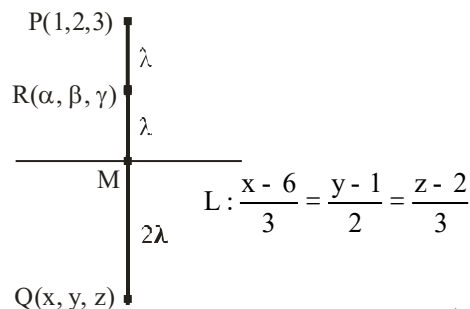
SECTION-B

1. Let the image of the point $P(1, 2, 3)$ in the line $L: \frac{x-6}{3} = \frac{y-1}{2} = \frac{z-2}{3}$ be Q . let $R(\alpha, \beta, \gamma)$ be a point that divides internally the line segment PQ in the ratio $1 : 3$. Then the value of $22(\alpha + \beta + \gamma)$ is equal to

Official Ans. by NTA (125)

Allen Overseas Ans. (125)

Sol.



Let M be the mid-point of PQ

$$\therefore M = (3\lambda + 6, 2\lambda + 1, 3\lambda + 2)$$

$$\text{Now, } \overrightarrow{PM} = (3\lambda + 5)\hat{i} + (2\lambda - 1)\hat{j} + (3\lambda - 1)\hat{k}$$

$$\therefore \overrightarrow{PM} \perp (3\hat{i} + 2\hat{j} + 3\hat{k})$$

$$\therefore 3(3\lambda + 5) + 2(2\lambda - 1) + 3(3\lambda - 1) = 0$$

$$\lambda = \frac{-5}{11}$$

$$\therefore M = \left(\frac{51}{11}, \frac{1}{11}, \frac{7}{11}\right)$$

Since R is mid-point of PM

$$22(\alpha + \beta + \gamma) = 125$$

2. Suppose a class has 7 students. The average marks of these students in the mathematics examination is 62, and their variance is 20. A student fails in the examination if he/she gets less than 50 marks, then in worst case, the number of students can fail is

Official Ans. by NTA (0)

Allen Overseas Ans. (0)

Sol. $20 = \frac{\sum_{i=1}^7 |x_i - 62|^2}{7}$

$$\Rightarrow |x_1 - 62|^2 + |x_2 - 62|^2 + \dots + |x_7 - 62|^2 = 140$$

$$\text{If } x_1 = 49$$

$$|49 - 62|^2 = 169$$

then,

$$|x_2 - 62|^2 + \dots + |x_7 - 62|^2 = \text{Negative Number}$$

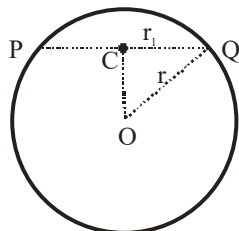
which is not possible, therefore, no student can fail.

3. If one of the diameters of the circle $x^2 + y^2 - 2\sqrt{2}x - 6\sqrt{2}y + 14 = 0$ is a chord of the circle $(x - 2\sqrt{2})^2 + (y - 2\sqrt{2})^2 = r^2$, then the value of r^2 is equal to

Official Ans. by NTA (10)

Allen Overseas Ans. (10)

Sol.



PQ is diameter of circle

$$S: x^2 + y^2 - 2\sqrt{2}x - 6\sqrt{2}y + 14 = 0$$

$$C(\sqrt{2}, 3\sqrt{2}), O(2\sqrt{2}, 2\sqrt{2})$$

$$r_1 = \sqrt{6}$$

$$S_1: (x - 2\sqrt{2})^2 + (y - 2\sqrt{2})^2 = r^2$$

Now in ΔOCQ

$$|OC|^2 + |CQ|^2 = |OQ|^2$$

$$4 + 6 = r^2$$

$$r^2 = 10$$

4. If $\lim_{x \rightarrow 1} \frac{\sin(3x^2 - 4x + 1) - x^2 + 1}{2x^3 - 7x^2 + ax + b} = -2$, then the

value of $(a - b)$ is equal to

Official Ans. by NTA (11)

Allen Overseas Ans. (11)

Sol. $\lim_{x \rightarrow 1} \frac{\sin(3x^2 - 4x + 1) - x^2 + 1}{2x^3 - 7x^2 + ax + b} = -2$

For finite limit

$$a + b - 5 = 0 \quad \dots(1)$$

Apply L'H rule

$$\lim_{x \rightarrow 1} \frac{\cos(3x^2 - 4x + 1)(6x - 4) - 2x}{(6x^2 - 14x + a)} = -2$$

For finite limit

$$6 - 14 + a = 0$$

$$\boxed{a = 8}$$

From (1) $\boxed{b = -3}$

Now $(a - b) = 11$

5. Let for $n = 1, 2, \dots, 50$, S_n be the sum of the infinite geometric progression whose first term is

n^2 and whose common ratio is $\frac{1}{(n+1)^2}$. Then the

value of $\frac{1}{26} + \sum_{n=1}^{50} \left(S_n + \frac{2}{n+1} - n - 1 \right)$ is equal to

Official Ans. by NTA (41651)

Allen Overseas Ans. (41651)

$$S_n = \frac{n^2}{1 - \frac{1}{(n+1)^2}} = \frac{n(n+1)^2}{(n+2)}$$

$$S_n = \frac{n(n^2 + 2n + 1)}{(n+2)}$$

$$S_n = \frac{n[n(n+2)+1]}{(n+2)}$$

$$S_n = n \left[n + \frac{1}{n+2} \right]$$

$$S_n = n^2 + \frac{n+2-2}{(n+2)}$$

$$S_n = n^2 + 1 - \frac{2}{(n+2)}$$

$$\text{Now } \frac{1}{26} + \sum_{n=1}^{50} \left[(n^2 - n) - 2 \left(\frac{1}{n+2} - \frac{1}{n+1} \right) \right]$$

$$= \frac{1}{26} + \left[\frac{50 \times 51 \times 101}{6} - \frac{50 \times 51}{2} - 2 \left(\frac{1}{52} - \frac{1}{2} \right) \right]$$

$$= 41651$$

6. If the system of linear equations

$$2x - 3y = \gamma + 5,$$

$\alpha x + 5y = \beta + 1$, where $\alpha, \beta, \gamma \in \mathbf{R}$ has infinitely many solutions, then the value of $|9\alpha + 3\beta + 5\gamma|$ is equal to

Official Ans. by NTA (58)

Allen Overseas Ans. (58)

Sol. $2x - 3y = \gamma + 5$

$$\alpha x + 5y = \beta + 1$$

Infinite many solution

$$\frac{\alpha}{2} = \frac{5}{-3} = \frac{\beta+1}{\gamma+5}$$

$$\alpha = \frac{-10}{3}, \quad 5\gamma + 25 = -3\beta - 3$$

$$9\alpha = -30, \quad 3\beta + 5\gamma = -28$$

$$\text{Now, } 9\alpha + 3\beta + 5\gamma = -58$$

$$|9\alpha + 3\beta + 5\gamma| = 58$$

7. Let $A = \begin{pmatrix} 1+i & 1 \\ -i & 0 \end{pmatrix}$ where $i = \sqrt{-1}$.

Then, the number of elements in the set

$$\{n \in \{1, 2, \dots, 100\} : A^n = A\} \text{ is}$$

Official Ans. by NTA (25)

Allen Overseas Ans. (25)

Sol. $A = \begin{bmatrix} 1+i & 1 \\ -i & 0 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 1+i & 1 \\ -i & 0 \end{bmatrix} \begin{bmatrix} 1+i & 1 \\ -i & 0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} i & 1+i \\ -i+1 & -i \end{bmatrix}$$

$$A^4 = \begin{bmatrix} i & 1+i \\ -i+1 & -i \end{bmatrix} \begin{bmatrix} i & 1+i \\ -i+1 & -i \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$$A^{4n+1} = A$$

$$n = 1, 5, 9, \dots, 97$$

$\Rightarrow$ total elements in the set is 25.

8. Sum of squares of modulus of all the complex numbers z satisfying $\bar{z} = iz^2 + z^2 - z$ is equal to

Official Ans. by NTA (2)

Allen Overseas Ans. (2)

Sol. $z + \bar{z} = iz^2 + z^2$

Consider $z = x + iy$

$$2x = (i+1)(x^2 - y^2 + 2xyi)$$

$$\Rightarrow 2x = x^2 - y^2 - 2xy \text{ and } x^2 - y^2 + 2xy = 0$$

$$\Rightarrow 2x = -4xy$$

$$\Rightarrow x = 0 \text{ or } y = \frac{-1}{2}$$

Case 1 : $x = 0 \Rightarrow y = 0$ here $z = 0$

Case 2 : $y = \frac{-1}{2}$

$$\Rightarrow 4x^2 - 4x - 1 = 0$$

$$(2x-1)^2 = 2$$

$$2x-1 = \pm\sqrt{2}$$

$$x = \frac{1 \pm \sqrt{2}}{2}$$

Here $z = \frac{1+\sqrt{2}}{2} - \frac{i}{2}$ or $z = \frac{1-\sqrt{2}}{2} - \frac{i}{2}$

Sum of squares of modulus of z

$$= 0 + \frac{(1+\sqrt{2})^2 + 1}{4} + \frac{(1-\sqrt{2})^2 + 1}{4} = \frac{8}{4} = 2$$

9. Let $S = \{1, 2, 3, 4\}$. Then the number of elements in the set $\{f : S \times S \rightarrow S : f \text{ is onto and } f(a, b) = f(b, a) \geq a \forall (a, b) \in S \times S\}$ is

Official Ans. by NTA (37)

Allen Overseas Ans. (37)

Sol. $(1, 1), (1, 4), (4, 1), (2, 4), (4, 2), (3, 4), (4, 3), (4, 4)$ – all have one choice for image.

$(2, 1), (1, 2), (2, 2)$ – all have three choices for image

$(3, 2), (2, 3), (3, 1), (1, 3), (3, 3)$ – all have two choices for image.

So the total functions $= 3 \times 3 \times 2 \times 2 \times 2 = 72$

Case 1 : None of the pre-images have 3 as image

Total functions $= 2 \times 2 \times 1 \times 1 \times 1 = 4$

Case 2 : None of the pre-images have 2 as image

Total functions $= 2 \times 2 \times 2 \times 2 \times 2 = 32$

Case 3 : None of the pre-images have either 3 or 2 as image

Total functions $= 1 \times 1 \times 1 \times 1 \times 1 = 1$

$\therefore$ Total onto functions $= 72 - 4 - 32 + 1 = 37$

10. The maximum number of compound propositions, out of $p \vee r \vee s$, $p \vee r \vee \sim s$, $p \vee \sim q \vee s$, $\sim p \vee \sim r \vee s$, $\sim p \vee \sim r \vee \sim s$, $\sim p \vee q \vee \sim s$, $q \vee r \vee \sim s$, $q \vee \sim r \vee \sim s$, $\sim p \vee \sim q \vee \sim s$ that can be made simultaneously true by an assignment of the truth values to p, q, r and s, is equal to

Official Ans. by NTA (9)

Allen Overseas Ans. (9)

Sol. If we take

p	q	r	s
F	F	T	F

The truth value of all the propositions will be true.